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ALTERNATIVE MODELS FOR EXCHANGE AND SPATIAL DISTRIBUTION

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Characterisation methods and the development of efficient field recovery procedures have now made possible the quantitative investigation of trade or distribution patterns in a detailed manner. The most obviously suitable subjects for such studies are classes of artefact, such as pottery or chipped stone, which are of relatively frequent occurrence at the sites where they do occur so that quantitative measures, in terms of frequency (whether absolute or relative) or quantity recovered per unit volume, can be established. The focus of investigation has thus moved beyond simply the identification of the source of origin for individual finds and hence the demonstration of contact. Instead the underlying regularities in the patterns observed are being sought, with the aim of understanding the mechanisms of exchange involved and hence of gaining an insight into the economic and social processes at work in the society in question.

It is the aim of this paper to draw attention to some of the regularities that are now becoming apparent and to ask to what extent we are justified in associating these with specific kinds of trade or exchange. (Exchange is here interpreted in the widest sense; indeed in the case of some distributions it is not established that the goods changed hands at all. Trade in this case implies procurement of materials from a distance, by whatever mechanism.)

In particular two different models for the traffic in goods are shown to produce distributions of the same form. Their coincidence raises questions of a more general nature.

The Law of Monotonic Decrement

When a commodity is available only at a highly localised source or sources for the material, its distribution in space frequently con-

forms to a very general pattern. Finds are abundant near the source, and there is a fall-off in frequency or abundance with distance from the source. This pattern is observed in many archaeological instances, and fall-off patterns of this kind are familiar to geographers. Frequency of occurrence declines with distance. That this should be so is not unduly surprising, since, in general, the transport of goods from a source requires the input of energy and, other things being equal, the greater the distance the greater the energy input required. The generality, that there is a fall-off in frequency or abundance with distance from source, shows sign of implying further and more interesting regularities. Moreover, departures from it are likely to be of interest and significance. It can be stated as follows:

In circumstances of uniform loss or deposition, and in the absence of highly organised directional (i.e., preferential, non-homogeneous) exchange, the curve of frequency or abundance of occurrence of an exchanged commodity against effective distance from a localised source will be a monotonic decreasing one.

This implies little more than that frequency decreases with increasing distance. The first proviso is an important one (see Ammerman *et al.* 1978). And we are dealing here with *effective* distance, which is not necessarily the same as the distance between points, since the ease of traversing terrain has to be considered. Deserts and mountains, acting as barriers or impediments, will increase effective distance. In certain circumstances rivers and sea will decrease it, depending always on the transport available. Effective distance may indeed be regarded as a measure of the energy required to move goods between two points, and it is crucially dependent upon the mode of transport available. The development of new travel technologies – the inception of long distance maritime travel or the use of the camel in desert lands – fundamentally alters effective distance.

The absence of directional exchange is a crucial point, and on the adequate definition of it rests the status of the law. For if directional trade were recognised solely by departure from the regularity, the 'law' would become a tautology. Directional exchange and concentration effects are discussed below.

The geographical literature about distance decay effects is already extensive (cf. Haynes 1974, Claeson 1968, Olsson 1965). It embraces many aspects of the spatial distribution of human interaction and offers stimulating insights for the archaeologist. But

while we stand to learn much from the rigour with which some geographers have approached their data, the models offered should not be accepted without some consideration of the underlying theory.

Various approaches have now been made toward a regression analysis of the fall-off with distance of traded commodities (cf. Renfrew *et al.* 1968, Hodder 1974). In each case some measure of abundance or frequency is plotted against distance from the source in question. The choice of the measure of abundance used is an important one. Naturally it is preferable to select a measure that is not massively biased by recovery techniques in the field, where so often a high frequency is a measure of intensity of archaeological activity rather than of real variations in the archaeological record itself. In operational terms the choice of an appropriate measure is crucial.

It is the possibility of reaching general conclusions of wider application that makes this field of research such an interesting one at present. Demonstrable regularities conforming to mathematically simple forms are still disappointingly few in archaeology, and their analysis raises new and interesting questions. For there is hope that many archaeological distributions from different times and places, and entirely independent in the circumstances of their formation, can be expressed in this way. If it is the case that these distributions share basic, simple properties, the same may be true also for the processes generating them. And the understanding of these cultural processes is the ultimate goal of our investigation.

Shape of Fall-off

Three principal classes of curves have, up to the present time, been considered relevant as possible approximations for the distance regressions observed. In each the variation of some measure of frequency or abundance I (whether expressed in occurrence per unit area or ratio of occurrence to that of some other material) at a point is related to the distance x of the point from the source locality:

$$I = f(x) \quad (1)$$

A. The Pareto Model

The first of these is a power function known as the Pareto model (see figure 1).

$$I = mx^{-k} \quad (2)$$

where k and m are constants. To those reared on Newtonian mechanics and the inverse square law, such a formulation seems a natural one to describe action at a distance. But there is no clear theoretical basis for the application of such an expression to human interactions, and geographers have in general found that it does not fit their data well.

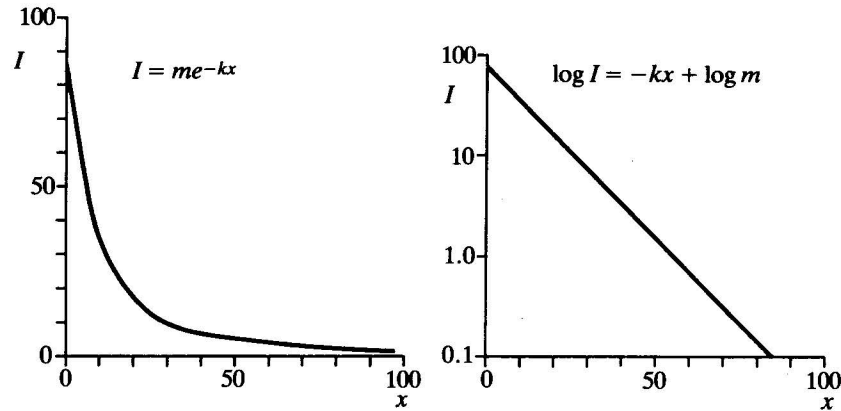


Figure 1. The Pareto model. On the left, interaction and distance are both on a linear scale; on the right, both are on a logarithmic scale.

To investigate such a relationship, logarithmic plotting may be used. For

$$\log I = -k \log x + \log m \quad (3)$$

so that when a power relationship holds, the logarithm of interaction plotted against the logarithm of distance on log-log paper will give a straight line with slope $-k$ and intercept $\log m$ on the ordinate. Wright (1970) has used such a plot for obsidian in the Near East.

B. Exponential Distance Decay

$$I = m \cdot e^{-kx} \quad (4)$$

$$\log I = \log m - kx \quad (5)$$

In this case e is the base of the natural logarithms, and fall-off is dependent on e raised to the power of the distance. When an exponential relationship holds, $\log$ (interaction) plotted against distance – not against the logarithm of distance – will give a straight line

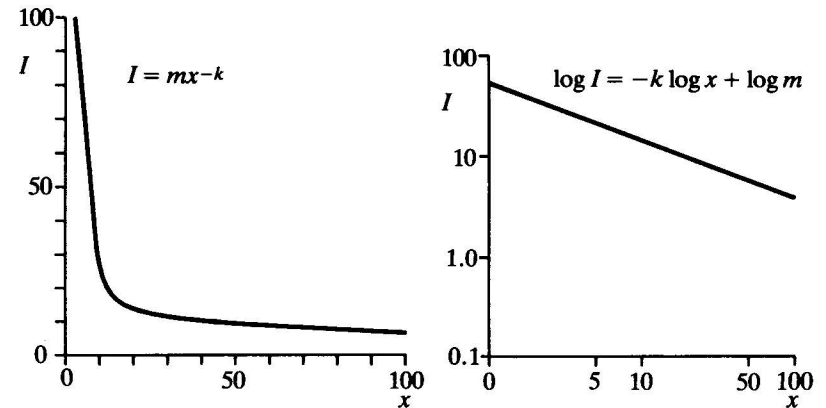


Figure 2. Exponential distance decay. On the left, interaction and distance are both on a linear scale; on the right, interaction is on a logarithmic scale, distance on a linear one.

of slope $-k$ and with intercept $\log m$ on the ordinate (see figure 2).

C. Gaussian Fall-Off

Since the work of Pearson and Blakeman (1906), Gaussian (normal) fall-off has been associated with the idea of random flights (see figure 3).

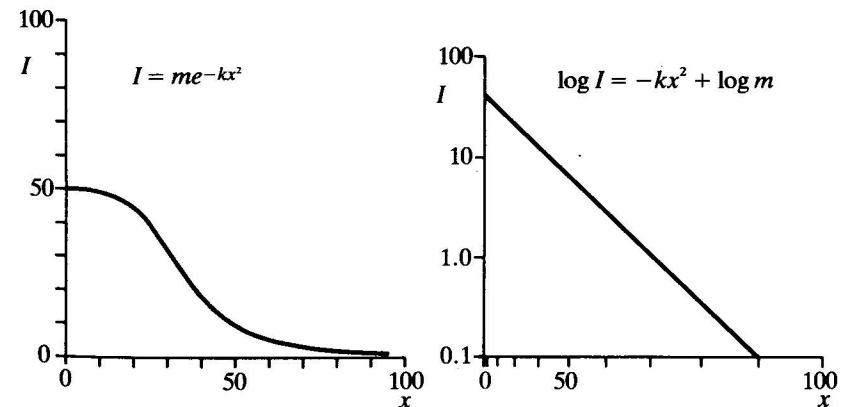


Figure 3. Gaussian fall-off. On the left, interaction and distance are both on a linear scale; on the right, interaction (on a logarithmic scale) is plotted against the square of distance.

$$I = m \cdot e^{-kx^2} \quad (6)$$

It follows that

$$\log I = \log m - kx^2 \quad (7)$$

So that if $\log I$ is plotted on the ordinate against x^2 on the abscissa, a straight line will result, of slope $-k$ and intercept $\log m$ on the ordinate. The idea of random flights was first applied to archaeological distributions by Hogg (1971) when considering the distributions of Iron Age coin types in Britain and the distance of the findspot from the issuing mint.

It should be noted that equations (4) and (6) are specific cases of the more general relationship

$$I = m \cdot e^{-kx^\alpha} \quad (8)$$

where α , the exponent, is a constant. Its value is 1 in the exponential case and 2 in the Gaussian.

Other fall-off patterns may yet be sought in the data; log normal, Pareto exponential, and square root exponential have been considered by geographers (Morrill 1963, Morrill and Pitts 1967).

Steepness of Fall-off

The shape of the fall-off curve is discussed further below. Clearly, in the expressions given, it depends first on whether a power or exponential relationship is chosen and secondly on the value of the constant k , which in equations (5) and (7) determines the slope of the line produced when the transformed variables are plotted.

It is in fact intuitively fairly obvious that some commodities will travel farther than others, and this has been confirmed in quantitative studies by geographers, demonstrating the greater travelling power of high-value goods.

This relates in some cases to the distinction between what I have called down-the-line exchange and prestige-chain exchange (Renfrew 1972; see pp.125–7 above).

The underlying idea is that the commodity or artefact has reached its destination as a result of a number of exchange transactions. The two models in question envisage a chain of exchange transactions by which the artefact finds its way from the source to the place where it finally enters the archaeological record. Four features of the latter were put forward (Renfrew 1972, 465–8).

That formulation fails to do justice to the complexity of the situation and the number of factors operating, nor does it follow

that high-value goods are necessarily prestige goods, which amount to what George Dalton calls 'primitive valuables' (by implication belonging to a different sphere of exchange), although this is often the case. Among the relevant factors governing the distance a given object travels are mean distance transported between exchange transactions (in prestige exchange, it is expected that the distances are greater); transportability (expressed as a ratio of value to weight and breakage rate in transit) and effective life, considering frequency of use, breakage rate in use, reuse-discard after breakage, loss-recovery rate, and deliberate burial.

A consideration of these factors – and no doubt there are others – makes it clear that 'high-value' commodities will have a smaller slope than others. The exchange of prestige goods involves at least two further considerations: namely, a restriction in the number of those for whom it is appropriate to have them, and the existence of 'spheres of conveyance' restricting the free exchange of one class of good for another. The exchange of *Spondylus* shells in neolithic southeast Europe may be one such instance. Hammond et al. have discussed the Maya jade trade in comparable terms, and Pires-Ferreira (1973, 257) has considered iron ore mirrors in Oaxaca in this way.

The Down-the-Line Model:

Linear Attenuation and Exponential Fall-off

In 1968, when working with obsidian data from the Near East (Renfrew *et al.* 1968, figure 2), I noticed that an approximately linear pattern was obtained by plotting the percentage of obsidian in the total chipped stone industry on a logarithmic scale (distance being plotted on a linear scale). As indicated above, this implies an exponential fall-off. A possible generating model was proposed as follows (cf. Renfrew 1972, 446).

Imagine a line of villages linked in a trading network in a linear manner and equally spaced a distance l apart. Each receives a supply of a particular commodity from its neighbour nearer the source, retains some for its own use (which ultimately finds its way into the archaeological record as debris, etc.), and passes the remainder on. If each village passes on a proportion k of what it receives (k being less than 1), then

- Village 1 at distance l passes on Nk
- Village 2 at distance $2l$ passes on Nk^2
- Village 3 at distance $3l$ passes on Nk^3
- Village n at distance nl passes on Nk^n

At distance x the amount passed on is $N \cdot k^{x/l}$. An exponential fall-off is generated by this series of exchange transactions.

The assumption of a chain of villages at uniform spacing is, however, a needlessly special case of a more general condition for exponential decay, namely a uniform distribution of population. Assume first that reduction in number of artefacts or quantity of material is proportional to the number or quantity left at the point in question, namely I . Assume further that this population is uniformly distributed and takes a constant fraction before passing the remainder on.

$$-dI \propto I dx$$

$$\int \frac{dI}{I} = -\int k \cdot dx \quad \text{where } k \text{ is constant}$$

$$\log I = -kx$$

$$I = I_0 e^{-kx} \quad \text{where } I_0 \text{ is constant} \quad (9)$$

This model makes no statement about a number of transactions but states simply that there is a continual attrition or attenuation proportional to the flow of goods at the point. Exponential distance decay has been well discussed by Haynes (1974). He is using a slightly different model, that of the journeys or trips from the source along a line away from the source.

The key condition here is that the reduction is proportional to the number or quantity at the point in question. If this assumption were modified so that the reduction were independent of the number left but dependent only on change in distance, we would have (in one dimension) simply a linear fall-off:

$$-dI \propto dx$$

$$\int -dI = \int k \cdot dx \quad \text{where } k \text{ is constant}$$

$$I - I_0 = -k \cdot x \quad \text{where } I_0 \text{ is constant}$$

$$I = I_0 - kx \quad (10)$$

Random Walk or Flight. Gaussian Fall-off

Pearson (1906) was faced with the problem of the infiltration of a species into a new habitat. He therefore conducted a mathematical analysis of the distribution of mosquitoes departing from a point source. For the archaeologist the analogy is that of transactions involving the transport of commodities, each period of ownership being the counterpart of a single flight, an exchange transaction

terminating one flight and starting another. When a large number of transactions (flights) are included, the situation is clear. Pearson's expression is

$$\phi(r^2) = \frac{N}{2\pi\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right) \quad (11)$$

where ϕ is the frequency of individuals in a small area distant r from the centre of dispersion, N is the number of individuals starting from that centre, and $\sigma^2 = 2nl^2$, where l is the length of each flight and n the number of random flights.

Although containing more constants, this is of the same form as our equation (6):

$$I = m \cdot e^{-kx^2}$$

since r is the equivalent of x .

When n is fairly small, the concentration is near the centre of dispersion. But when n is much larger, the distribution is more widely dispersed.

This generative model indicates clearly how a Gaussian distribution may occur in many cases analogous to the random flights of mosquitoes. It should be noted, however, that in this case the term 'random' refers only to the direction of flight. The model is highly nonrandom in other respects, for each mosquito is constrained to undertake precisely n flights, each of length exactly l . In most real cases, however, the length of flight is not fixed but could be given by a frequency distribution. More seriously for the archaeological cases likely to be encountered, the objects in question certainly do not undertake the same number of flights or walks. The archaeological record is in fact the sum of the random walk functions for $n = 1, n = 2$, and so on up to a high value for n . Once again a frequency distribution could be constructed for the number of flights. A further necessary modification may be to allow the source to produce further individuals as the successive flights of existing individuals take place.

The essential point here remains that a large number of uncoordinated events will, in certain circumstances, produce a coherent, quantifiable fall-off curve in this way. The world offers many examples of such behaviour – Brownian motion is one – and there are analogies here for the diffusion of innovation as well as for the exchange of goods.

Attenuation in Two Dimensions

In an earlier section, fall-off along a line was discussed for the situation in which reduction of goods at each point is proportional to the quantity/frequency of goods at the point in question. The result was an exponential fall-off. It is interesting to extend this assumption to exchange in two dimensions, making the same assumptions as in the one-dimensional case. This, then, is the general situation of diffusion or attenuation in two dimensions. As Dr R. H. Dean has kindly indicated to me, the result is once again a Gaussian distribution. Assume

1. That the reduction at a point is proportional to the number or quantity left, I .
2. That the population is uniformly distributed and takes a constant fraction

$$-dI \propto I \cdot 2\pi x \, dx$$

$$\text{i.e. } \int -dI/I = \int 2kx \, dx \quad \text{where } k \text{ is constant} \quad (12)$$

$$\log I/I_0 = -kx^2$$

$$\text{or } I = I_0 e^{-kx^2} \quad (13)$$

This is an important result, since it takes the same form as equation (6), although the generating model is a very different one.

In order to complete the examination of random walk, uniform attenuation, and constant fall-off in one and two dimensions, two other cases should be considered at this point. Again I owe their formulation to Dr Dean.

(1) *Reduction independent of number left; two dimensions.* Here we are making the same assumption that led, for one dimension, to equation (9). The reduction in number or quantity is independent of the number or quantity left.

$$-dI \propto 2\pi x \, dx$$

$$\text{i.e. } \int -dI = \int 2kx \, dx \quad \text{where } k \text{ is constant}$$

$$I - I_0 = kx^2 \quad (14)$$

This is parabolic.

(2) *Random walk in one dimension.* A particle or object has an equal probability of going to the right or to the left at each decision. After N steps let it be displaced n steps to the right. What is the probability of this? The total number of choices is 2^N . The 'successful' number (i.e., the number of ways of producing a shift of n) is

$${}^N C_{(N+n)/2} = \frac{N!}{\left(\frac{N+n}{2}\right)! \left(\frac{N-n}{2}\right)!}$$

Therefore the probability is

$$\frac{N!}{\left(\frac{N+n}{2}\right)! \left(\frac{N-n}{2}\right)!} 2^{-N}$$

If there are many steps, this becomes

$$\text{Probability} = \left(\frac{2}{\pi N}\right)^{1/2} \exp -\left(\frac{n^2}{2N}\right) \quad (15)$$

This is again a Gaussian expression.

We are thus in a position to summarize the shape of fall-off curves obtained for each of the three models in one and two dimensions, assuming that the distribution of population is uniform (see table 1).

Table 1. The shape of the curves resulting from distance fall-off effects from a point source, along a line, and on a plane, with a uniform distribution of population.

Model	One dimension	Two dimensions
Reduction in number independent of number left	linear (10)*	parabolic (14)
Reduction in number proportional to number left	exponential (9)	Gaussian (13)
Random walk	Gaussian (15)	Gaussian (11)

* Figures in parentheses refer to equations in the text

Equifinality and the Choice of Models

In earlier sections two different models for prehistoric exchange are discussed that might explain fall-off effects. The first represents exchange transactions in two dimensions in terms of random flights, where the direction of travel is random although other features (length of flight, number of flights) are fixed. The second postulates exchange transactions in two dimensions originating from a point source, the quantity of commodity retained at any location being a given constant proportion of that which reaches it. The remarkable circumstances is that the shape of the two distributions is identical: it is Gaussian.

Dr Dean has kindly pointed out another case. We have assumed throughout that the distribution of population is uniform. To modify this assumption varies the results. For example in equation 12, we have

$$-dI \propto I \cdot 2\pi x \, dx$$

But if the population density falls off as $1/x$ from the centre of distribution or dispersal, this becomes

$$-dI \propto I \cdot 2\pi \, dx$$

which again gives rise to equation (9), the exponential case.

The circumstance in which different initial conditions and different processes lead to the same end-product in the archaeological record suggests at first the notion of equifinality in general systems theory. There is no difficulty, of course, in defining the formation processes and preservation process of the archaeological record as a system. As Kast and Rosenzweig (1972, 23) express it:

In physical systems there is a direct cause and effect relationship between the initial conditions and the final state. Biological and social systems operate differently. The concept of *equifinality* says that final results may be achieved with different initial conditions and in different ways. This view suggests that the social organisation can accomplish its objectives with varying inputs and varying internal activities. Thus the social system is not restrained by the simple cause and effect relationship of closed systems.

Equifinality is a fascinating concept because of the great scope of the cases to which it is applicable (cf. Von Bertalanffy 1950, 157f.). But I am not sure that it is applicable here, or rather that it relates to the behaviour of the culture system, which is the focus of our interest. For there is no suggestion that the culture system in the cases under discussion did in fact reach a final, steady state to which the term equifinality could be applied. In a sense, I suppose, the formation processes did reach a steady state, namely the state in which we find the archaeological record. But this realisation does not help us in the choice between two models, both producing the same patterning in the data. A more effective resolution must be to seek for independent tests for the different assumptions on which those models rest.

Comparisons of Fall-off Data

The most systematic investigation to date of distance decay data from archaeological contexts has been undertaken by Ian Hodder (1974). In his thoughtful and wide-ranging survey he has for the first time collected sufficient instances to make the distinctions discussed above of more than academic interest. At the same time, the treatment given above of the models generating the different patterns puts some of these real cases in a new light.

Hodder used the very general expression, equation (8) above, in the form

$$\log I = a - bD^\alpha + c \quad (16)$$

His method was to use a computer program to calculate the best fit regression line, performing the calculation for several values of α in succession (varying in steps of 0.1 from $\alpha=0.1$ to $\alpha=2.5$). As we have seen, in the exponential case, $\alpha=1$; in the Gaussian case, $\alpha=2$.

Hodder reported that his data fell into two groups, and such a division is reflected in the work of geographers studying distance decay effects. In the first group α was around 1 or greater. He cites Anatolian obsidian ($\alpha=0.9$), Dobunnic coins ($\alpha=1.3$), Roman fine ware from Oxford and Hampshire ($\alpha=1.0$ and 1.3), neolithic pottery from Cornwall ($\alpha=1.6$), and neolithic axes ($\alpha=2.5$) (cf. Hodder 1974, figure 17).

In the second group are materials whose fall-off shows a very low α value, between 0.1 and 0.6. Examples are Roman roofing tiles, Roman coarse pottery, and Middle Bronze Age palstaves.

In the discussion it must be remembered that the value of α determines the shape of the curve and hence the nature of the model. It is not merely some expression for the value of a constant in an equation.

The first group, with value of α around 1, suggests some conformity with the exponential decay situation. Higher values of α , around 2, indicate Gaussian fall-off. It is not correct to suggest that a figure near unity ($\alpha=0.9$) for Anatolian obsidian may be seen as the result of a random walk process. The very thrust of Hodder's elegant regression analysis must be to show that, for a random walk, $\alpha=2$.

The analysis tends to suggest exponential rather than Gaussian fall-off for some commodities, and our models indicate that this implies some linear patterning. Attenuation down a line produces exponential decay, while attenuation in two dimensions or random

walk along a line produce Gaussian fall-off ($\alpha = 2$). Down-the-line trade of this kind has to be seen in terms of trading chains and trading networks.

Hodder's second group suggests to me something very different. When α is near zero, neither exponential decay nor random walk can apply at all. He had documented that many of the goods in question are common, bulky, and supplied in a limited area: for instance, Roman roofing tiles near Cirencester. In some cases the slope of the regression line is also very steep. This leads me to suggest that the models discussed above, which are framed in terms of repeated transaction, do not apply in such cases. In some of them the pattern may not be very far from the linear one described above (equation (10)) or even the parabolic (equation (14)).

I suggest that this is *supply zone* behaviour (Renfrew *et al.* 1968, 327), where we are dealing with a pattern arising largely from single journeys. In many cases the user is travelling direct to the source or manufacturing centre, or the producer is travelling with his goods direct to the purchaser. The result is an extreme localisation in the distribution of the product that is not in general handed on in subsequent transactions. There is a radius beyond which the specific product is very rarely found, and this radius is usually the length of a single journey that the producer or user will normally undertake to sell it or to fetch it. In some cases there may be an intermediate transaction at a local market. But in general, within the supply zone, the goods are distributed by means of a single exchange transaction. This is mode 2 of a recent formulation of trading types (Renfrew 1965, 42).

This is to be contrasted with *contact zone* behaviour, where commodities are worth exchanging beyond the limits of the supply zone (although the distance travelled may be limited by the effect of competing sources – cf. p.150ff.). Often they are distributed by down-the-line exchange. As Hodder well brings out, these are usually either more desirable, or easier to transport, or both.

Concentration Effects and Directional Trade

We have discussed so far distributions in which the exchange is taking place from hand to hand in a homogeneous population. That is to say, no one individual or place is particularly distinguished from any other. In these circumstances any location will receive less of a commodity than its neighbour nearer to the source.

Not infrequently, however, this principle, the law of monotonic decrement, is violated, and a more distant source obtains not only

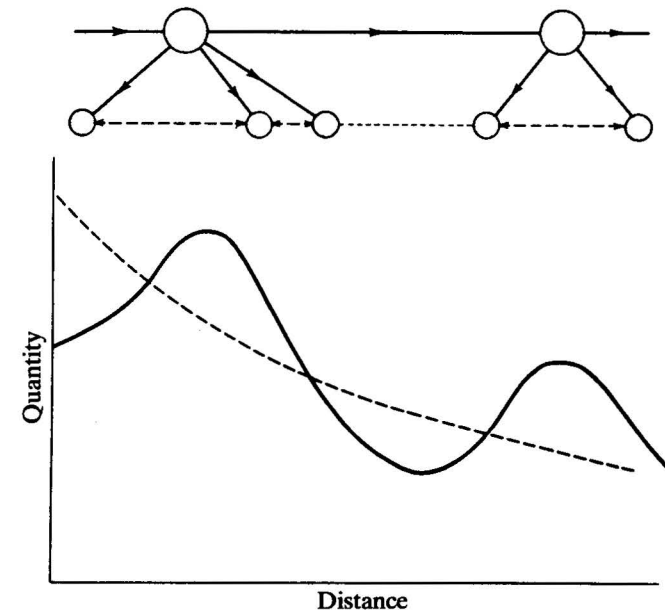


Figure 4. Directional trade: the effect of central places (above) on the fall-off. The dashed curve (below) shows down-the-line exchange: exponential fall-off generated by exchange in a linear chain (quantity and distance on linear scales).

more of the commodity but a greater *per capita* frequency of it. This can only mean that it is being supplied preferentially (with respect to its neighbours), and it is here that the term directional trade is appropriate.

In the first place, it is relevant to see in this the development both of larger centres of population and of central places. A large centre of population, whatever its nature, will attract a large absolute quantity of any commodity (see p.126ff.). But this will not of itself guarantee any larger quantity *per head* of population.

The concept of 'central place' implies, however, more than simply larger size. The central place is a locus for exchange activity, and more of any material passes through it (per head of population) than through a smaller settlement (see figure 4).

The hierarchy of settlement is here accompanied by a hierarchy of exchange activity. Suppliers from a distance bring their goods first to the central place, and they are disseminated from the central

place to smaller localities. In other words, the central place is used for break of bulk. The effect here is that, in terms of supply, the central place is nearer to the source than are lower-order localities supplied by it, even if these may in reality be geographically closer to the source.

If the archaeological record is formed in proportion to the quantity of material handled, the central place will show a greater frequency than its population alone would warrant, since it is acting as a supply centre for its hinterland.

In practice, however, the effect is sometimes more marked than this. For accompanying the hierarchy of places is a hierarchy of persons. And those persons in the upper parts of the hierarchy have preferential access to the material and will accumulate greater quantities, per head, of commodities they desire.

Here, then, are two concentration effects. The first arises from the central place or market function of a centre, with the break of bulk of the commodity. The second arises from preferential access of prominent or wealthy individuals who are generally located at central places. Both these concentration effects result in a greater frequency (per head of population or per cubic metre of excavated soil) than would otherwise be the case. Were it not for them, the total of a commodity at a large population centre would be greater than in surrounding settlements, but the frequency would not.

Directional trade is to be recognised by indications of break-of-bulk operations (storehouses, organisational systems, waste materials) and by signs of preferential supply in large quantity. In favourable cases, both of these will result in a higher frequency of finds and hence in a departure from the monotonic decrease effect encountered with nondirectional trade.

This argument has been used (Renfrew and Dixon 1977) to explain the decline in the concentration of obsidian at rural sites in the Near East in the middle and late neolithic period, it being suggested that the emergence of central places was actually impairing the supply of obsidian at lower-order localities. Sidrys has elegantly documented the concentration effect operating for obsidian at Classic Maya ceremonial centres.

Competing Sources

Care is needed in the use of gravity models for the prediction of traded materials as reflecting interaction with centres of population. For, as implied in the last section, the larger population attracting a larger quantity of material need not result in any greater frequency,

expressed in frequency per head or per cubic metre of soil.

The impact of a second source of the same commodity, however, is important. Bradley (1971) suggests that the asymmetrical spread of various pottery products, as well as of obsidian, in the Near East may be interpreted in these terms. An equally interesting case has arisen in the obsidian distribution of the Sardinia, Lipari, and Parmarola sources of southern Italy (Hallam *et al.* 1976).

If we use a modification of the Law of Retail Gravitation (Reilly 1931), this impact can be explained. Let the material traded have a desirability factor or attractiveness A_1 . Then source S_1 will supply a quantity Q_1 directly proportional to its attractiveness A_1 and inversely proportional to some function of the distance D_1 from the source. (In this formulation the square of the distance is used.) That is

$$Q_1 \propto A_1/D_1^2$$

When dealing with two sources

$$Q_1/Q_2 = A_1/A_2 \times D_2^2/D_1^2 \quad (17)$$

And if we take the locus of equal interaction, $Q_1 = Q_2$, and

$$A_1/A_2 = D_1^2/D_2^2 = k^2 \quad (18)$$

where k is the ratio of the "attractiveness" of the sources. If d is the effective distance of separation of the sources, then it is easy to show that the locus of equal interaction is a circle of radius $kd/(1-k^2)$ with a centre lying on the line joining the landfalls and displaced a distance $k^2d/(1-k^2)$ from the least attractive source (see figure 5).

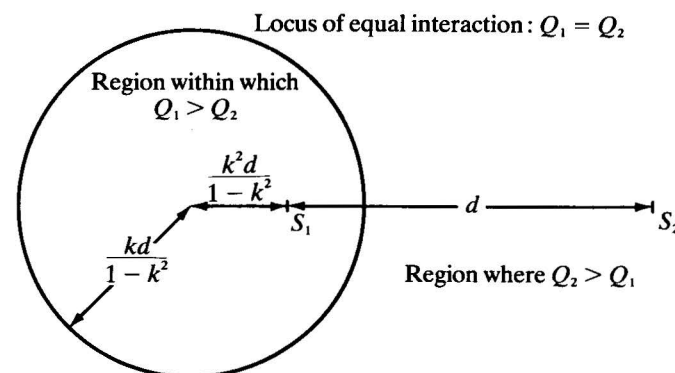


Figure 5. The locus of equal interaction for two competing sources S_1 and S_2 (after Hallam *et al.* 1976).

It is quite possible for two sources a distance x apart to give rise quite independently to fall-off patterns, which taken together might suggest the behaviour of competing sources discussed here. The two patterns superimposed, as if they were contemporary, will give a first approximation to that predicted by the Law of Retail Gravitation. It is therefore necessary to distinguish the effect of straight-forward, independent distance fall-off from that of sources simultaneously in competition. A 'test' of the model, apparently supporting its application, would be of little worth. Moreover, it is probably inappropriate to use an inverse square relationship for distance, as explained in much of the preceding discussion. Until these problems have been carefully thought out, applications and modifications of the Reilly formula should perhaps be viewed with suspicion.

Modes of Exchange

A recent attempt (see p.120, figure 10) to identify the spatial correlates of various specific modes of trade, in the vocabulary of Polanyi, must be viewed with some sobriety in the light of the foregoing discussion.

Central place redistribution and central place market exchange are spatially identical (although concentration effects are facilitated by redistributive control or by tax).

These two modes should, however, be distinguishable from symmetrical, homogeneous, reciprocal exchange networks. I suggest that a useful field for further study will be the range of organisational devices through which the law of monotonic decrement is circumvented by human exchange systems, particularly those that are hierarchically structured. [1977]

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